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THE MOLECULAR BRANCHING RATIO METHOD FOR CALIBRATION OF  
OPTICAL SYSTEMS IN THE VACUUM ULTRAVIOLET

by

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ABSTRACT

The intensity distribution of bands belonging to six molecular band systems is discussed with special emphasis on their usefulness for intensity calibration of optical systems in the vacuum ultraviolet ( $1000\text{\AA} < \lambda < 3000\text{\AA}$ ). The theory of molecular band intensities is outlined and the technique of measuring the spectral response curve is described. Several methods for establishing an absolute intensity calibration are discussed. The molecular band systems which are described are:  $\text{H}_2(\text{B}^1\Sigma_u^+ - \text{X}^1\Sigma_g^+)$ ,  $\text{H}_2(\text{C}^1\Pi_u - \text{X}^1\Sigma_g^+)$ ,  $\text{N}_2(\text{a}^1\Pi_g - \text{X}^1\Sigma_g^+)$ ,  $\text{CO}(\text{A}^1\Pi - \text{X}^1\Sigma^+)$ ,  $\text{NO}(\text{A}^2\Sigma^+ - \text{X}^2\Pi_r)$ , and  $\text{NO}^+(\text{A}^1\Pi - \text{X}^1\Sigma^+)$ .

## INTRODUCTION

Until recently, calibration of optical systems in the vacuum ultraviolet ( $1000 < \lambda < 3000\text{\AA}$ ) was extremely difficult. With the exception of the atomic branching ratio technique, the available techniques were not suited to in-situ calibration of an optical system as a "black-box."<sup>1,2</sup> Furthermore, the method of atomic branching ratios gave only a few scattered calibration points over a wide wavelength range and required two optical systems, one to calibrate the long-wavelength atomic line intensity (typically He 5015 $\text{\AA}$ ) and one for use in the vacuum ultraviolet (typically He 584 $\text{\AA}$ ). For completeness, we mention some of the other calibration procedures which were available or suggested: (1) "double-monochromator technique" used with a thermopile detector, sodium salicylate detector, or photo-ionization detector, (2) synchrotron emission, used as a known source with a calculable spectral distribution, and (3) delayed-coincidence atomic line fluorescence<sup>3,4</sup> (e.g. H(3s-2p) 6563 $\text{\AA}$  and H(2p-1s)1216 $\text{\AA}$ ). These methods suffered from one or more of the following problems: (1) limited wavelength range, (2) impracticability of in-situ measurements, (3) uncertainty of polarization effects, and (4) self-absorption of atomic lines terminating on the ground electronic state. These problems were largely overcome by the recently developed "molecular-branching-ratio technique".

The extension of the branching ratio technique to molecular band intensities was suggested independently by McConkey<sup>5</sup> and Aarts and de Heer<sup>6,7</sup>

McConkey found good agreement between the spectral response (3000 - 4500Å) measured with an NBS quartz-iodine lamp and the spectral response determined from intensity measurements on the  $N_2(C^3\Pi_u - B^3\Pi_g)$  second positive group. Aarts and de Heer measured the relative intensities of bands (1400-2600Å) belonging to the  $CO(A^1\Pi - X^1\Sigma^+)$  fourth positive group and the  $CO^+(B^2\Sigma^+ - X^2\Sigma^+)$  first negative group. Although they recognized that  $R_e$  might not be constant for the systems, Aarts and de Heer assumed constant  $R_e$  for the purposes of deriving a spectral response curve and demonstrating the usefulness of the technique. This early work<sup>5,6,7</sup> demonstrated the feasibility of the method, however, the reliability was unknown since the variation of electronic transition moment had not been accurately measured or calculated for any band system ( $\lambda < 3000\text{\AA}$ ) except  $N_2(a^1\Pi_g - X^1\Sigma_g^+)$ .

The first quantitative, detailed treatments of the technique in the vacuum ultraviolet were performed independently by Mumma and Zipf<sup>8,9</sup> ( $N_2(a-X)$ , and  $CO(A-X)$ ) and by Becker et al.<sup>10</sup> ( $H_2$  and  $HD(B-X)$ ). Further advances have been made by Poland and Broida<sup>11</sup> ( $NO(A-X)$ ), Stone and Zipf<sup>12,13</sup> ( $NO^+(A-X)$  and  $H_2(C-X)$ ) and Aarts and de Heer<sup>14</sup> ( $NO^+(A-X)$ ). In this paper we review the theory and the molecular band systems which may be used for calibration purposes in the vacuum ultraviolet ( $1000\text{\AA} < \lambda < 3000\text{\AA}$ ). The method is useful for wavelengths longer than 3000Å as well. Band systems longward of 3000Å have not been included in this review since standard lamps are routinely used for calibration purposes there ( $\lambda > 2600\text{\AA}$ ).

# THEORY

Consider two molecular electronic states which are connected by an electric dipole-allowed transition. The volume emission rate (photons/cm<sup>3</sup> sec 4πSr) will be given by

$$\beta_{v',v''} = n_{v'} A_{v',v''} \quad (1)$$

Where  $N_{v'}$  is the number density (cm<sup>-3</sup>) in level  $v'$  and  $A_{v',v''}$  is the transition probability (sec<sup>-1</sup>). If the emission is incident on an optical system (monochromator + windows + detector) which has a spectral sensitivity  $R(\lambda)$ , when the measured counting rate (counts/sec) for a band ( $v', v''$ ) is given by

$$S_{v',v''} = GR(\lambda_{v',v''}) \beta_{v',v''} \quad (2)$$

"G" is a geometrical function involving the acceptance solid angle of the optical system, source characteristics, monochromator slit settings, etc. G is kept constant for observations of a given band system and is thus of no importance in determining the relative spectral response.

$A_{v',v''}$  is the transition probability<sup>15</sup> and is given by

$$A_{v',v''} = \frac{64\pi^4}{3h} q_{v',v''} \nu_{v',v''}^3 R_e^2(v', v''), \quad (3)$$

where  $q_{v',v''}$  is the vibrational overlap integral (Franck-Condon-factor FCF),  $\nu_{v',v''}$  is the wave-number (cm<sup>-1</sup>), and  $R_e$  is the electronic transition moment. The lifetime of the vibrational level,  $v'$ , is  $\tau_{v'}$ , is  $\tau_{v'} = (A_{v'})^{-1}$  where

$$A_{v'} = \sum_{v''} A_{v',v''} \quad (4)$$

and the molecular branching ratio is defined to be

$$B. R. = A_{v',v''}/A_{v',.} \quad (5)$$

The relative intensities of bands belonging to a  $v''$  progression ( $v'$  constant) are independent of  $n_{v'}$  (eqn's 1, 2, 3) and depend only on the branching ratios, so that

$$\frac{S_{v',v''_1}}{S_{v',v''_2}} = \frac{R(\lambda_{v',v''_1}) q_{v',v''_1} v_{v',v''_1}^3 R_e^2(v',v''_1)}{R(\lambda_{v',v''_2}) q_{v',v''_2} v_{v',v''_2}^3 R_e^2(v',v''_2)} \quad (6)$$

This means that the relative spectral response of the optical system can always be extracted from measurements of the relative counting rates for a given  $v''$  progression, independent of the way in which the various  $v'$  are populated. Thus, effects such as cascade, quenching, self absorption (except for bands terminating on  $v''=0$ ), and excitation mechanism (e.g. exchange interaction vs. direct excitation by electron impact) have no effect on the measured relative spectral response. Hence, the method is readily used in discharge systems, controlled electron beam experiments, and photo-excitation sources (here, polarization effects may be important<sup>11</sup>). The results for different  $v''$  progressions are best internormalized by requiring a least-squares-fit of the data to a third-order expression in  $\lambda$  for  $R(\lambda)$  (see Mumma and Zipf<sup>8</sup> for further details). A requirement for using eqn. 6 is that  $R_e^2$  be known for bands ( $v',v''$ ). A frequently successful approach is to express  $R_e$  in terms of the r-centroid,  $\bar{r}_{v',v''}$ <sup>16,17,18</sup>.

Under certain circumstances (e.g. monoenergetic electron impact,  $E > 100$  ev) cascade contributions to the excited state may be negligible and

the excitation cross sections,  $\sigma_{ov'}$ , given by <sup>15,19a</sup>

$$\sigma_{ov'} \propto f_{ov'} \lambda_{ov'}^3 A_{ov'}$$

$$\sigma_{ov'} \propto q_{ov'} R_e^2(\bar{r}_{ov'}), \quad (7)$$

where  $f_{ov'}$  is the absorption f-value. For lifetimes,  $\tau_{v'}$ , which are sufficiently short so that the molecule essentially radiates in-situ, the equilibrium number density,  $n_{v'}$ , is given by

$$\frac{dn_{v'}}{dt} = 0 = J n_o \sigma_{ov'} - n_{v'} A_{v'},$$

or

$$n_{v'} = \frac{J n_o \sigma_{ov'}}{A_{v'}}, \quad (8)$$

where J represents the electron beam current density. Then the relative counting rates in the band system are given by (eqn's 1, 2, 3, 7 and 8)

$$\frac{S_{v_1'v_2''}}{S_{v_3'v_4''}} = \frac{R(\lambda_{v_1'v_2''})}{R(\lambda_{v_3'v_4''})} \frac{q_{ov_1'}}{q_{ov_3'}} \frac{R_e^2(\bar{r}_{ov_1'})}{R_e^2(\bar{r}_{ov_3'})} \frac{(qv^3)_{v_1'v_2''}}{(qv^3)_{v_3'v_4''}} \frac{R_e^2(\bar{r}_{v_1'v_2''})}{R_e^2(\bar{r}_{v_3'v_4''})} \frac{A_{v_3'}}{A_{v_1'}}. \quad (9)$$

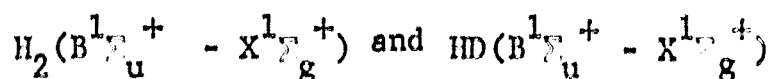
Usually the electronic transition moment varies across the band system, but when it is constant eqn's (6) and (9) reduce to especially simple form. Accurate Franck-Condon-Factors and r-centroids are available for many systems. Albritton, Schmeltekopf, and Zare's RKR calculations are especially useful since their tables include  $q$ ,  $\bar{r}$ , and  $qv^3$ . <sup>20</sup>

Excitation of molecular band systems by electron impact at moderately high energies is expected to produce essentially unpolarized radiation



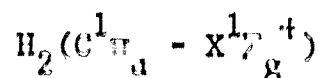
since many closely-spaced rotational levels are usually excited (exception,  $H_2$ , e.g. see Ref. 21). While a particular rotational line may show polarization effects, one expects that rotational averaging yields a net polarization near zero for the band. This is the case for the NO  $\gamma$  bands.<sup>11</sup> However, when NO  $\gamma$  line fluorescence is excited by level-crossing-spectroscopy, the resultant rotational lines are polarized. If the monochromator has a wavelength-dependent-polarization, optical calibration by polarized molecular line fluorescence can give misleading results.

# DISCUSSION OF SPECIFIC BAND SYSTEMS



Becker, Fink, and Allison<sup>10</sup> have excited single rotational levels in the  $\text{H}_2$  B-state ( $v'=3, J'=1$ ) and the HD B-state ( $v'=3, J'=2$ ) by absorption of the Ar 1066.66 $\text{\AA}$  resonance line. The emitted (B-X) radiation consists of one P-Branch line and one R-Branch line for each transition ( $3, v''$ ). Thus the  $\text{H}_2$  "many-line" spectrum is reduced to an easily used subset of 28 lines (Table I). The transition probabilities have been calculated ab initio by Allison and Dalgarno and their results were confirmed by the independent calculation of Julienne.<sup>22a</sup> Becker et al.<sup>22b</sup> showed that a calibration curve established using the Lyman lines (1100 - 1650  $\text{\AA}$ ) was in good agreement with similar measurements (1325 - 1800 $\text{\AA}$ ) on the  $\text{N}_2(\text{a}^1\Pi_g - \text{X}^1\Sigma_g^+)$  Lyman-Birge-Hopfield band system in the range of overlap (1325 - 1650  $\text{\AA}$ ).

In application, the emitted line intensities are much brighter from HD than from  $\text{H}_2$ , since the Ar 1066  $\text{\AA}$  line is in closer resonance with the HD transition and the statistical weight for the ground state level is more favorable. The useful wavelength range is smaller than that indicated in Table V since the (3,0) lines are subject to self absorption and the (3,3), (3,12), and (3,13) lines are quite weak in emission. The practical range over which the Lyman line fluorescence may be used for calibration purposes is thus 1112 - 1638  $\text{\AA}$ .

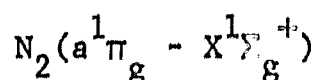


Aarts and de Heer<sup>23</sup> and Carriere and de Heer<sup>24</sup> first attempted to use the Werner bands for calibration purposes. Their intensity measurements were made at  $4\text{\AA}$  resolution which was later shown to be inadequate to eliminate band overlap,<sup>12</sup> giving rise to a pronounced dip in the resultant calibration curve around  $1200\text{\AA}$ .

Stone and Zipf<sup>12</sup> recently have investigated the use of Werner bands for calibration in the VUV. They find that a spectral resolution of  $0.44\text{\AA}$  is required to eliminate most of the problems produced by overlapping lines. The lines of the P and R branches are known to be subject to strong perturbations due to mixing of the  $\text{B}'^1\Sigma_u^+$  and  $\text{C}^1\pi_u$  states. The Q branch lines are not subject to this perturbation since the symmetries of the C rotational levels which generate the Q-branch are different from the symmetries of the corresponding B' levels. The Q1 lines are mainly free from overlap by P and R branch lines. Stone finds that their observed intensities are in close agreement ( $\pm 3\%$ ) with the theoretical intensities calculated by two methods: (1) Equation 6 and the ab initio transition probabilities of Allison and Dalgarno<sup>22</sup>, (2) Equation 9 and the appropriate Franck-Condon factors and electronic transition moments.

Insufficient wavelength resolution leads to erroneous results for the relative spectral response of an optical system when calibrated with emission intensities of the Werner bands. Several earlier attempts at calibration using these bands were performed at  $4\text{\AA}$  resolution. This is now understood to give spurious structure in the resultant calibration curve and has unfortunately resulted in some confusion in the literature on dissociative excitation cross sections.

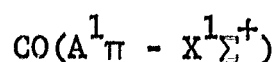
The relative emission intensities for Q1 lines have been calculated using equation 6 and the transition probabilities of Allision and Dalgarno and are presented along with the appropriate wavelengths in Table II. To date, only Q1 lines in the range 1100-1250Å have been experimentally verified to follow these intensity relations. The prospective user is cautioned to check for overlap by P and R Branch lines.



The Lyman-Birge-Hopfield band system of  $N_2$  consists of compact (FWHM  $\leq 2\text{\AA}$ ) single headed bands (1275 - 2100Å) which are readily excited by electron impact. The electronic transition (a-X) is forbidden by electric dipole interaction and proceeds mainly by magnetic dipole interaction although there is some electric quadrupole contribution as well.<sup>25</sup> McEwen<sup>26</sup> was the first to quantitatively investigate the emission intensities of these bands and he established that  $R_e$  was constant to within  $\pm 20\%$ . McEwen's intensity calibration was based on the constant quantum yield of sodium salicylate over the wavelength range in question. Subsequent investigations by numerous authors have found no variation in  $R_e$ . Holland<sup>27</sup> observed the emission intensities with an optical system which had been calibrated using the double-monochromator-technique and a thermopile detector. Lassettre<sup>28</sup> used the electron-energy-loss method to show that the excitation cross sections,  $\sigma_{ov'}$ , followed the Franck-Condon factors for  $v'=0$  through  $v'=12$ . Mumma<sup>8,29</sup> used NI branching ratios

to verify that  $R_e$  was constant. Ajello<sup>30</sup> measured the band emission intensities using an optical system which was calibrated using the double-monochromator method and a sodium salicylate detector. A curve of growth analysis<sup>31</sup> also indicated a constant  $R_e$ , however, these data covered a very limited range of r-centroids and the results are not indicative of the whole band system.

The lifetime of the  $a^1\pi_g$  state is  $\sim 1.60 \times 10^{-4}$  seconds.<sup>25</sup> Thus the excited molecule can experience many collisions and can travel several inches before radiating. The vibrational population of the a-state may not be given by eqn. 8. However, the relative emission intensities for bands belonging to a given progression ( $v'$  constant) will still be given by eqn. 6. In Table III, we present the bandhead wavelengths and relative emission intensities for the LBH system. The a-state vibrational distribution is strongly dependent on the experimental excitation conditions. When the system is excited by monoenergetic electrons ( $E_e > 100$  eV) at low pressure ( $< 10^{-4}$  Torr), the vibrational populations follow the weighting factors  $q_{ov'}/A_{v'}^8$ . Under these conditions the relative emission intensities (normalized to the 3,0 band) are obtained by multiplying the tabulated values by the appropriate weighting factor. By contrast, in another experiment, the Lewis-Rayleigh afterglow of  $N_2$  produced  $N_2(a^1\pi_g)$  with  $v'=0,1,2,3$  highly populated but  $v'=4,5$  only weakly so<sup>10</sup>. In practice, the user must exercise caution when analyzing the observed spectrum in order to correctly account for the effects of band overlap. A wavelength resolution of  $1\text{\AA}$  or better is highly recommended.



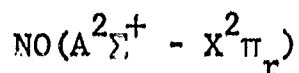
The fourth positive group of CO consists of single headed compact bands (1400 - 2200 Å) which are degraded toward the red. The bands are readily excited by electron impact in CO<sup>9,32,33</sup> due to the large electronic oscillator strength for this transition (absorption f-value = 0.19)<sup>9,34</sup>. Cascade into the A state has been shown to be negligible for moderately high electron energies<sup>32,35</sup> (> 100 eV). The equilibrium vibrational distribution of the A state is thus given by eqn. 8. There are some references (e.g Ref. 36) in the early literature which claim that the electronic transition moment,  $R_e$ , is constant for this system. This early work suffered from inadequate intensity calibration procedures which led to incorrect conclusions. In fact,  $R_e$  varies quite strongly with the r-centroid. Mumma et al.<sup>9</sup> have determined the dependence of  $R_e$  on  $\bar{r}$  using an optical system which was calibrated with the molecular branching ratio method ( $N_2$  LBH system) and the atomic branching ratio method (NI multiplets). They found

$$R_e \propto 1.0 - 0.6 \bar{r}_{v'v''} \quad (10)$$

No information regarding the coefficient of the second order term could be obtained since the data were adequately plotted by a straight line. This dependence was independently confirmed by the electron-energy-loss spectra of Lassetre et al.<sup>34</sup> who found exactly the same functional form for  $R_e$ . Since Lassetre's experiment did not use optical techniques the exact agreement constituted a direct and independent confirmation of the optical calibration techniques developed by Mumma and Zipf<sup>8</sup>. It also provided indirect confirmation of the constancy of  $R_e$  for the  $N_2$  LBH

system. Recent lifetime data of Imhof and Read<sup>37</sup> indicate that an inclusion of the quadratic term is necessary to reproduce the observed small variation of lifetime with  $v'$ . However, the quadratic term is expected to have only a small effect on the calculated intensities for bands with recenteroids in the range  $1.05 < \bar{r} < 1.35$ , since  $R_e$  is well represented by eqn. 12 in that range. These bands lie to the left of the dashed curve in Table IV.

The absolute transition probabilities have been calculated using the FCF's and r-centroids of Albritton et al.<sup>20</sup> and the above expression for  $R_e(\bar{r})$  (eqn. 10). The relative intensities were then calculated using eqn. 9 and were normalized to the (2,0) band.<sup>9</sup> The results are given in Table IV along with the bandhead wavelengths. The lifetimes of the levels  $v'$  are typically  $\sim 10$  nsec,<sup>35,37</sup> thus the limits of the emitting region correspond to the electron beam limits. For monoenergetic electron impact ( $> 100$  eV) Table IV gives the relative volume emission rates directly. When the vibrational distribution can not be described by eqn. 8, the relative volume emission rates of bands belonging to different progressions cannot be described by Table IV. However, the relative intensities of bands within a given progression ( $v'$  constant) will still be given by the appropriate row in Table IV.



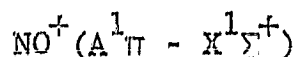
The NO(A-X)  $\gamma$  band system occurs in the wavelength range 1900 - 3400Å. The emission bands form four heads (doublet - double headed) and are degraded to the violet. The system has been studied extensively both theoretically<sup>20,38,39</sup> and experimentally.<sup>11,40,41</sup> Franck-Condon factors and r-centroids have been calculated assuming both Morse<sup>38</sup> and RKR<sup>20,39</sup> potential functions. The recent RKR calculation of Albritton et al.<sup>20</sup> yields Franck-Condon factors which are in close agreement with the calculation of Nicholls<sup>38</sup> which was based on Morse potential functions. The calculations of Flinn et al.<sup>39</sup> do not give correct relative intensities for the bands in emission, a fact which was first noted by Callear et al.<sup>40</sup> and confirmed by Poland and Broida.<sup>11</sup> The first quantitative study of these bands in emission was performed by Robinson and Nicholls.<sup>41</sup> They concluded that  $R_e$  varied strongly but this was later shown to be incorrect by several authors.<sup>11,40</sup> Callear's comparison of the observed emission intensities with Flinn's FCF's should be disregarded since Flinn's FCF's have been superceded by Albritton's. However, Callear also compared the observed emission intensities with Nicholls' Morse FCF's, which we have already noted are in good agreement with Albritton's. This comparison showed that  $R_e$  was nearly constant for  $1.00 < \bar{r} < 1.10\text{Å}$ . Poland and Broida showed that  $R_e$  was constant to within 10% over the band system. We have therefore taken Albritton's intensity factors and wavelengths as representative of the relative emission intensities of these bands (Table V). They may be used for calibration purposes in the range 2100-2600Å.

Poland and Broida excited the NO  $\gamma$  system by absorption of the Xe continuum which results in extensive fluorescence ( $v'=0, 1, 2, 3$ ). They also used level-crossing-spectroscopy to excite specific



K' levels in the  $A^2\Sigma^+$  state.<sup>11,42</sup>  $Cd^+ 2144\text{\AA}$  radiation<sup>11</sup> was used to excite  $v'=1$ ,  $K'=13$  (two spin levels were excited) and the  $Zn 2138.56\text{\AA}$  resonance line<sup>42</sup> was used to excite  $v'=1$ ,  $K'=23$  and 29. The resultant A-X rotational line radiation ( $1, v''$ ) was found to be highly polarized, unlike the case when the extended band system was excited. One must exercise considerable caution in using the line fluorescence of NO to avoid polarization dependent effects in the measured monochromator spectral response. The extended band emission (excited by Xe continuum absorption) showed no polarization ( $< 2\%$ ).

The measured lifetimes of the  $A^2\Sigma^+$  levels are approximately independent of  $v'$ , but the levels are fed by cascade as well as being excited directly.<sup>43</sup> The reported lifetimes range from 200 nsec to 100 nsec.<sup>35,43</sup>



The  $NO^+(A^1\Pi)$  state is readily excited by electron impact ionization and photo-ionization of NO. Several groups<sup>13,14,44</sup> have recently investigated the emission intensities of the  $NO^+(A-X)$  bands which were excited by monoenergetic electron impact. Aarts and de Heer<sup>14</sup> and Stone and Zipf<sup>13</sup> find that  $R_e$  varies to second order in  $\bar{r}$  while Mentall and Morgan<sup>44</sup> are able to fit their observed intensities assuming only first order dependence on  $\bar{r}$ . All three groups used photoelectric detection and established their relative intensity calibrations by using the molecular branching ratio method for  $N_2$ . However, Stone and Zipf calibrated their system using the HD(B-X) line fluorescence method as well. In addition, they followed the computer-least-squares-fit method in establishing their calibration curve.

Finally, photoelectron spectroscopy<sup>45</sup> yields relative level cross sections,  $\sigma_v/\sigma_o$ , and a variation of  $R_e$  with  $\bar{r}$  for the  $\text{NO}(X^2\Pi) \rightarrow \text{NO}^+(A^1\Pi)$  transition which agree well with Stone's results. For these reasons, we accept Stone and Zipf's (equivalently, Aarts and de Heer's) functional form for  $R_e(\bar{r})$  for the  $\text{NO}^+(A-X)$  bands and their values for the level cross sections,  $\sigma_v$ . The relative emission intensities were calculated by eqn 9. They are given in Table VI and apply for electron energies in excess of 100 eV.

In practice, the  $\text{NO}^+(A-X)$  system is simple to use for calibration purposes since the problem of overlapping bands is not nearly so severe, as with  $\text{N}_2(\text{LBH})$  or  $\text{CO}^+(\text{A-X})$ . However, the wavelength range (1300-1600Å) is somewhat limited.

# ABSOLUTE MEASUREMENTS

The relative spectral response of an optical system may be established over a wide wavelength range by intensity measurements on the above band systems. The spectral response may be made absolute over the wavelength range to which it applies by determining the absolute detection efficiency at one wavelength, corresponding to establishing a value for  $G$  in eqn. 2.

At sufficiently high impact energies, the excitation cross section,  $\sigma_{v'}$ , in the Bethe approximation is given by<sup>19b</sup>

$$\sigma_{v'} = \frac{4\pi a_o^2 R^2}{E_{e1}} \frac{f_{ov'}}{E_{ov'}} \ln(4CE_{e1}/R) \quad (11)$$

where  $a_o$  is the first Bohr radius,  $R$  is the Rydberg energy,  $E_{ov'}$  is the excitation energy,  $f_{ov'}$  is the absorption oscillator strength,  $C$  is a constant, and  $E_{e1}$  is the energy of the incident electron. In the absence of cascade into level  $v'$ , quenching, or excitation transfer, the emission cross section of the  $(v', v'')$  band is given by

$$\sigma_{v'v''} = \frac{A_{v'v''}}{A_{v'}} \sigma_{v'}, \quad (12)$$

and thus depends on  $R_e$  through eqn. 3. Aarts and de Heer<sup>32</sup> established that the  $CO(A^1\Pi)$  state was not populated by cascade and used eqn's 11 and 12, along with the  $f$ -values of Lassettre and Skerbele<sup>34</sup> to establish absolute cross sections for the  $(0,1)$  band of  $CO(A^1\Pi - X^1\Sigma^+)$ . However, they assumed constant  $R_e$  which was later shown to vary quite strongly with  $\bar{r}$  by Mumma et al.<sup>9</sup> thus affecting the cross section  $\sigma_{v'v''}$  through the branching ratio in eqn. 12. Using the correct branching ratio, the

absolute emission cross section (Bethe theory) is

$$\sigma_{01} = 0.488 \times 10^{-18} \text{ cm}^2, E_{e1} = 500 \text{ eV} \quad (13)$$

This value for  $\sigma_{01}$  may be used to establish an absolute spectral response at  $1597\text{\AA}$ , the band head wavelength of the (0,1) band. The principle of using the absorption oscillator strength and the Bethe theory to establish absolute cross sections has recently been applied in the extreme vacuum ultraviolet as well by van Raan ( $\lambda < 1164\text{\AA}$ ) using noble gas resonance lines.<sup>46</sup>

The absolute response may also be established by measurements of the emission intensity of Lyman alpha radiation,  $\text{HI } 1216\text{\AA}$ , which is produced by electron impact dissociative excitation of  $\text{H}_2$ .<sup>47</sup> This cross section has been placed on an absolute scale by comparison with the cross section for exciting Lyman alpha by electron impact on atomic hydrogen, which was normalized to the Born approximation above 300 eV. At 100 eV, the value of the dissociative excitation cross section is

$$\sigma(1216) = 1.2 \times 10^{-17} \text{ cm}^2 \pm 11\%. \quad (14)$$

The error reflects the fact that the dissociative excitation cross section is related to the theoretical value of the direct excitation cross section by experiment. A wavelength resolution of  $\sim 1\text{\AA}$  is required to separate the Lyman alpha line from neighboring lines of the  $\text{H}_2(\text{C-X})$  Werner bands.

## CONCLUSION

The molecular branching ratio technique for calibration of monochromators has experienced rapid development in recent years. We have reviewed the state-of-the-art and critically discussed six band systems which may be used to establish the spectral response of an optical system over the wavelength range (1000-3000Å). We have also described how the relative spectral response may be placed on an absolute scale. Using these techniques the absolute spectral response can be established within  $\pm 10\%$  over a broad range of wavelengths. Thus absolute intensity calibrations may now be established with relative ease throughout a wavelength region where no simple, convenient, in-situ calibration techniques existed before the development of the molecular-branching-ratio method.

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TABLE I. Wavelengths and Transition Probabilities of  $H_2$  and  
 $HD (B^1\Sigma_u^+ \rightarrow X^1\Sigma_g^+)$  Lines.

$H_2$					$HD$			
Band	Line	$\lambda$ (Å)	$A_{\nu'J'\nu''J''}$ ( $\times 10^8 \text{ s}^{-1}$ )	Relative Intensity	Line	$\lambda$ (Å)	$A_{\nu'J'\nu''J''}$ ( $\times 10^8 \text{ s}^{-1}$ )	Relative Intensity
3,0	R(0)	1062.8	0.336	0.141	R(1)	1066.7	0.256	0.131
	P(2)	1066.8	0.705	0.296	P(3)	1071.8	0.411	0.210
3,1	R(0)	1112.0	0.759	0.319	R(1)	1109.7	0.792	0.405
	P(2)	1116.2	1.518	0.637	P(3)	1114.9	1.215	0.622
3,2	R(0)	1162.7	0.305	0.128	R(1)	1153.9	0.652	0.334
	P(2)	1167.1	0.555	0.233	P(3)	1159.3	0.924	0.473
3,3	R(0)	1214.9	0.046	0.019	R(1)	1199.2	0.028	0.014
	P(2)	1219.4	0.125	0.052	P(3)	1204.8	0.021	0.011
3,4	R(0)	1268.4	0.546	0.229	R(1)	1245.6	0.333	0.170
	P(2)	1273.0	1.110	0.466	P(3)	1251.4	0.565	0.289
3,5	R(0)	1322.8	0.232	0.097	R(1)	1292.9	0.615	0.315
	P(2)	1327.5	0.401	0.168	P(3)	1298.8	0.884	0.453
3,6	R(0)	1377.7	0.093	0.039	R(1)	1340.8	0.062	0.032
	P(2)	1382.5	0.240	0.100	P(3)	1346.8	0.053	0.027
3,7	R(0)	1432.6	0.600	0.251	R(1)	1389.2	0.293	0.150
	P(2)	1437.4	1.205	0.506	P(3)	1395.2	0.521	0.267
3,8	R(0)	1486.8	0.132	0.055	R(1)	1437.5	0.670	0.343
	P(2)	1491.6	0.196	0.082	P(3)	1443.6	0.966	0.495
3,9	R(0)	1539.3	0.325	0.136	R(1)	1485.4	0.075	0.038
	P(2)	1543.9	0.779	0.327	P(3)	1491.5	0.059	0.030
3,10	R(0)	1588.6	1.187	0.498	R(1)	1532.2	0.407	0.208
	P(2)	1593.3	2.382	1.000	P(3)	1538.2	0.754	0.386
3,11	R(0)	1634.0	0.542	0.227	R(1)	1577.2	1.284	0.658
	P(2)	1638.0	0.955	0.400	P(3)	1583.0	1.952	1.000
3,12	R(0)	1672.7	0.0071	0.0029	R(1)	1619.5	0.768	0.393
	P(2)	1676.1	0.0054	0.0022	P(3)	1625.0	1.018	0.521
3,13	R(0)	1702.6	0.0054	0.0022	R(1)	1657.9	0.076	0.039
	P(2)	1705.2	0.0131	0.0054	P(3)	1662.9	0.075	0.038

TABLE II. Wavelengths and Relative Intensities for Q1 Lines of the

 $N_2(C^1\Pi_u - X^1\Sigma_g^+)$  Werner Band System

$v'$	$v''=0$	1	2	3	4	5	6	7	8	9	10	11	12	13
0	319 1009.9*	809 1054.1	974 1099.5	386 1146.0	98 1193.3	12 1241.2								
1	780 986.9	551 1029.1	13 1072.3	759 1116.5	1000 1161.3	437 1206.7	76 1252.1	4 1297.2						
2	745 966.2	49 1006.6	411 1047.9	270 1090.0	126 1132.8	905 1175.9	711 1219.0	168 1261.7	10 1303.4					
3	475 947.5	21 986.3	307 1026.0	10 1066.3	358 1107.2	11 1148.3	463 1189.4	728 1230.0	227 1269.6	12 1307.6				
4	245 930.6	86 968.1	84 1006.2	119 1045.0	61 1084.2	142 1123.6	105 1162.9	151 1201.7	580 1239.5	231 1275.7	8 1309.4			
5	114 915.7	93 951.9	6 988.8	102 1026.3	2 1064.0	101 1101.9	15 1139.7	125 1177.0	30 1213.5	413 1249.0	193 1282.9	2 1315.0	1 1344.8	
6	50 902.2	67 937.3	2 273.1	43 1009.3	27 1045.8	16 1082.4	54 1118.8	2 1154.8	82 1189.9	3 1224.0	289 1256.6	134 1287.3		
7	21 890.1	40 924.3	8 959.1	11 994.3	28 1029.7	1069.1	28 1100.4	12 1135.1	14 1169.0	39 1201.9	214 1233.3	68 1262.9	8 1290.3	1315.3

\* Wavelengths after E. J. Stone, private communication.

TABLE III. Wavelengths and Relative Emission Intensities for the  
 $N_2(a^1\Pi_g-X^1\Sigma_g^+)$  Lyman-Birge-Hopfield System.

$v''=0$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
$v'$																		
0	215 1450	683 1501	1000 1555	896 1612	552 1672	247 1736	83 1805	21 1878	4 1956									
1	665 1416	1000 1464	371 1515	2 1570	337 1627	624 1688	518 1752	266 1821	95 1895	24 1973	5 2057							
2	1000 1384	513 1430	16 1479	463 1530	325 1585	2 1642	211 1703	458 1768	389 1838	195 1911	65 1990	16 2074	3 2165					
3	1000 1354	61 1398	338 1444	275 1493	15 1545	313 1600	183 1658	1 1719	196 1785	340 1854	254 1928	114 2007	34 2092	8 2182				
4	1000 1325	35 1368	493 1412	2 1459	325 1508	133 1560	64 1616	293 1674	88 1736	30 1801	246 1871	310 1945	189 2025	72 2109	19 2200	4 2297		
5	1000 1299	348 1339	315 1382	210 1427	308 1474	47 1523	353 1576	25 1631	184 1690	268 1752	14 1818	119 1888	333 1963	296 2042	143 2127	47 2218	10 2315	1 2420
6	1000 1273	926 1312	45 1353	654 1396	20 1441	470 1489	88 1539	257 1592	282 1648	17 1707	352 1769	178 1835	14 1906	298 1981	430 2060	276 2145	106 2236	4 2438



TABLE V. Wavelengths and Relative Emission Intensities of the  
 $\text{NO}(A^2\Sigma^+ - X^2\Pi) \gamma$  Band System.

	$v''=0$	1	2	3	4	5	6	7
$v'$								
0	721 2265.5	1000 2366.0	786 2474.2	463 2590.9	228 2717.0	100 2853.8	40 3002.5	15 3164.8
1	1000 2151.3	274 2241.8	--- 2338.7	147 2442.7	238 2554.4	206 2675.0	132 2805.2	71 2946.4
2	1000 2049.5	53 2131.5	420 2218.8	173 2312.2	--- 2412.1	63 2519.4	141 2634.5	145 2758.7
3	820 1958.1	1000 2032.8	193 2112.2	163 2196.6	393 2286.6	150 2382.7	--- 2485.5	56 2595.7

TABLE VI. Bandhead Wavelengths and Relative Intensities  
of the  $\text{NO}^+(\text{A}^1_\pi - \text{X}^1_\Sigma^+)$  Baer-Mischer Band System.

	$v''=0$	1	2	3	4	5
$v'$						
0	192 1368.3	641 1413.7	1000 1461.4	983 1511.8	676 1564.9	349 1621.1
1	256 1339.7	466 1383.1	281 1428.8	29 1476.9	33 1527.6	145 1581.0
2	150 1313.0	127 1354.7	8 1398.5	26 1444.6	51 1493.0	15 1544.0
3	52 1288.2	15 1328.3	3 1370.4	14 1414.5	2 1461.0	3 1509.8